GET READY

Before learning about combinations and permutations, practice using the Fundamental Counting Principle.

A food truck offers 3 appetizers, 4 main dishes, and 2 desserts. Customers choose exactly one appetizer, one main dish, and one dessert to build a combo meal.

Read the scenario, then use the response box provided to enter the total number of different combo meals possible. Include the multiplication expression used to find the answer.

How many different combo meals are possible? Include the multiplication expression used.

Type answer

You will use this foundational multiplication idea as you explore more complex counting methods today.

LEARN

When a counting problem involves choosing items from a group, the arrangement of the chosen items sometimes matters and sometimes does not. Knowing which situation you are in determines which counting method you should use.

Select each tab to learn how permutations and combinations differ. As you read, pay attention to the signal words in each example. You can use them to identify which method a problem calls for.

Permutations: Order Matters | Combinations: Order Does Not Matter

Suppose a class of 10 students needs to select a President, a Vice President, and a Treasurer. Each of the three roles is different, so choosing Amir for President and Bea for Vice President creates a different outcome than choosing Bea for President and Amir for Vice President. Situations where the order or role assigned to each choice changes the outcome are called permutations.

Permutations: Order Matters | **Combinations: Order Does Not Matter**

Now suppose that same class of 10 students needs to select 3 students to serve on a hallway decorating committee, with no distinct roles assigned. Choosing Amir, Bea, and Cho for the committee produces the same group no matter which name is listed first. Situations where the order of selection does not change the outcome are called combinations.

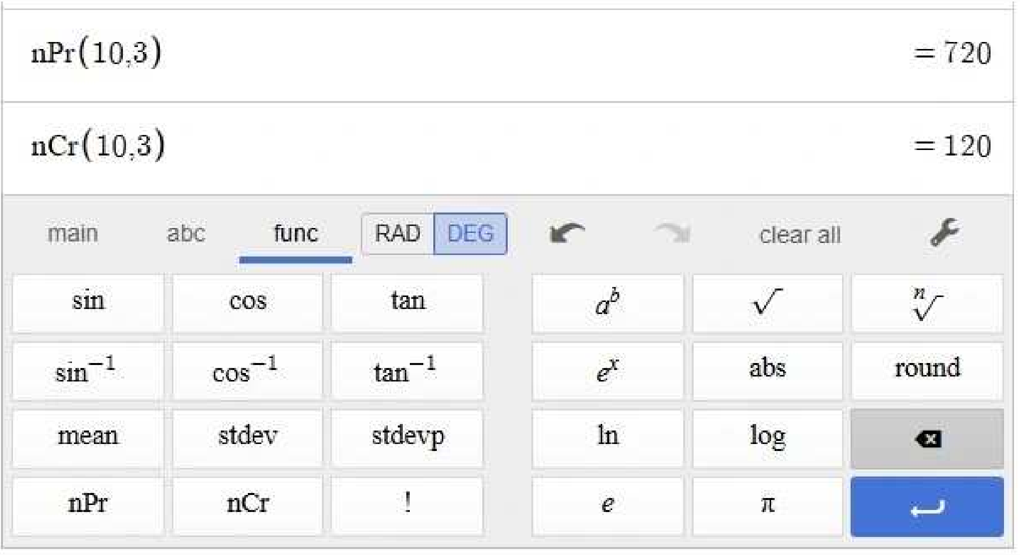
The number of possible outcomes shrinks when moving from a permutation to a combination of the same items, because a combination treats every different ordering of the same group as a single outcome instead of several separate outcomes.

Permutation signal words: rank, order, arrange, position, first/second/third place, distinct roles

Combination signal words: group, team, committee, selection, subset, no distinct roles

Each counting situation has a formula, and a calculator function that applies it quickly.

Select each tab to review the formula and calculator steps for that topic.



Permutations | Combinations | Calculator Steps | Comparing the Results

The permutation formula is ${}_nP_r = \frac{n!}{(n-r)!}$, where n is the total number of items and r is the number of items being arranged.

Example: choosing a President, Vice President, and Treasurer from 10 students uses ${}_{10}P_3 = \frac{10!}{(10-3)!} = 10 \cdot 9 \cdot 8 = 720$ possible outcomes.

Permutations | Combinations | Calculator Steps | Comparing the Results

The combination formula is ${}_nC_r = \frac{n!}{r!(n-r)!}$, where n is the total number of items and r is the number of items being chosen.

Example: choosing a 3-person committee from 10 students uses ${}_{10}C_3 = \frac{10!}{3!(10-3)!} = 120$ possible outcomes.

Permutations | Combinations | Calculator Steps | Comparing the Results

On the Desmos scientific calculator, both functions are entered using the func button rather than typed from the keyboard.

- For a permutation, tap the **func** button and select **nPr**. Then type the total number, a comma, and the number being arranged: **10,3** and press enter to get **nPr(10,3) = 720**.
- For a combination, tap the **func** button and select **nCr**. Then type the total number, a comma, and the number being chosen: **10,3** and press enter to get **nCr(10,3) = 120**.

You can use the Desmos scientific calculator directly in your browser to try this yourself.

Permutations | Combinations | Calculator Steps | Comparing the Results

${}_{10}P_3 = 720$ and ${}_{10}C_3 = 120$. Dividing 720 by 120 equals 6, which is $3! = 3 \cdot 2 \cdot 1$. This shows that each group of 3 people can be arranged in 6 different orders, which is exactly why the permutation total is 6 times larger than the combination total for the same group of people.

Probability compares the number of favorable outcomes to the total number of possible outcomes:

$$\text{probability} = \frac{\text{favorable outcomes}}{\text{total possible outcomes}}$$

Combinations often calculate both parts of this ratio. One combination finds the total number of possible groups, which becomes the denominator. A second combination finds the number of ways to select the specific favorable group, which becomes the numerator.

A problem that combines outcomes from more than one group into a single probability, such as this one, is called a compound event.

Select each section to reveal how this process applies to a marble-draw problem.

Step 1: Set Up the Scenario +

A bag contains 5 red marbles and 3 blue marbles, for a total of 8 marbles. Four marbles are drawn at random without replacement. The goal is to find the probability of drawing exactly 2 red marbles and 2 blue marbles.

Step 2: Find the Total Possible Outcomes (Denominator) +

The denominator counts every possible group of 4 marbles that could be drawn from the 8 total marbles, regardless of color. Since the order the marbles are drawn does not change the group, this uses a combination: ${}_8C_4 = 70$.

Step 3: Find the Favorable Outcomes (Numerator) +

The numerator counts only the groups that match the favorable outcome: exactly 2 red marbles and 2 blue marbles. This requires two combinations multiplied together: choosing 2 of the 5 red marbles, ${}_5C_2 = 10$, and choosing 2 of the 3 blue marbles, ${}_3C_2 = 3$. Multiplying these together gives $10 \times 3 = 30$ favorable groups.

Step 4: Calculate the Probability +

Dividing the favorable outcomes by the total possible outcomes gives $\frac{30}{70}$, which simplifies to $\frac{3}{7}$. This is the probability of drawing exactly 2 red marbles and 2 blue marbles.

THINK

To determine which counting method to use, ask yourself the "Swap Question."

Imagine you select Person A and then Person B. Now, imagine swapping their order: you select Person B and then Person A.

Does the swap create a different outcome or give them different responsibilities?

- If YES, the order matters. Use a permutation.
- If NO, it is the exact same result. Use a combination.

Read both scenarios, then consider which counting method, permutation or combination, applies to each one and why.

Scenario A: Talent Show Rankings

A talent show features 12 performers. Judges will award 1st place, 2nd place, and 3rd place trophies. Each of these three placements is distinct from the others.

Scenario B: Study Group Selection

The same 12 performers will select 3 students to form a study group for an upcoming exam. All 3 study group members share the same role.

For Scenario A, awarding 1st place, 2nd place, and 3rd place, explain why this situation is counted using a permutation rather than a combination.

Type answer

Explain why the total number of possible outcomes shrinks from Scenario A to Scenario B, even though the same 12 performers and the same group size of 3 are involved.

Type answer

When tackling complex compound probability problems, break the word problem into three distinct chunks before using your calculator:

1. Identify the Grand Total: How many total items exist, and how many are you pulling in total? Write this as your denominator.
2. Isolate the Groups: Split the items into their specific categories (e.g., red marbles vs. blue marbles).
3. Match the Pulls: Write a combination for each specific category based on what you want to pull. Multiply these combinations together in your numerator.

A bag contains 5 red tokens and 7 blue tokens. Four tokens are randomly selected at the same time.

Which fraction represents the probability of drawing exactly 2 red tokens and 2 blue tokens?

Select one

1. $\frac{{}_5C_2 \times {}_7C_2}{{}_{12}C_4}$
2. $\frac{{}_5P_2 \times {}_7P_2}{{}_{12}P_4}$
3. $\frac{{}_5C_2 + {}_7C_2}{{}_{12}C_4}$
4. $\frac{{}_{12}C_4}{{}_5C_2 \times {}_7C_2}$

Correct

This fraction pairs a combination for each token color in the numerator with the total combination for all 4 tokens in the denominator, which matches how a compound probability is structured when the tokens are drawn at the same time.

Incorrect

Consider whether each part of the fraction uses a combination rather than a permutation, whether the two favorable groups should be multiplied rather than added, and whether the denominator accounts for all 4 tokens drawn from the full set of 12.

A study group has a bag containing 6 chocolate granola bars and 9 fruit granola bars. They plan to grab 5 bars at once and want to find the probability of grabbing exactly 3 chocolate bars and 2 fruit bars. A classmate proposes this fraction:

$$\frac{{}_6P_3 \times {}_9P_2}{{}_{15}P_5}$$

Explain what is mathematically wrong with this fraction and how each part should be adjusted.

Type answer

TRY IT

A student is choosing 4 friends from a group of 8 to take to a concert. Which method should be used to find the total number of possible groups?

Select one

1. $_{8}P_{4}$

This expression treats each group of 4 friends as if a different order produces a different outcome. Reconsider whether the order you pick your friends changes who ends up going to the concert.

2. $_{8}C_{4}$

This expression treats every group of 4 friends as one outcome regardless of the order they were picked, which fits a situation where friends fill no distinct roles.

3. 8×4

This expression multiplies the total pool by the number chosen rather than counting groups of 4 at once. Revisit how a single selection of several friends together should be counted.

4. $8!$

This expression arranges all 8 friends in order rather than selecting a smaller group of 4. Revisit how many friends are actually being chosen for the concert.

A local art show has 12 paintings submitted. The judges will award a 1st, 2nd, and 3rd place ribbon. How many different ways can the ribbons be awarded?

Select one

1. 1,320

This value comes from treating 1st place, 2nd place, and 3rd place as three distinct positions, which fits how these ribbons are awarded.

2. 220

This value comes from treating the three ribbon positions as if they held no distinct roles. Reconsider whether receiving 1st place differs from receiving 3rd place.

3. 36

This value is much smaller than the total number of ways three distinct ribbons can be awarded among 12 paintings. Revisit how many choices remain for each ribbon position in turn.

4. 12

This value only accounts for one ribbon position rather than all three. Revisit how many ribbon positions still need to be filled.

A deck of cards has 10 green cards and 10 yellow cards. A hand of 5 cards is drawn. Which expression represents the probability of drawing exactly 3 green cards and 2 yellow cards?

Select one

1. $\dfrac{_{10}C_{3} + {_{10}C_{2}}}{_{20}C_{5}}$

This expression adds the two favorable groups together rather than pairing them within one hand. Revisit how the Fundamental Counting Principle combines separate groups into a single outcome.

2. $\dfrac{_{10}P_{3} \times {_{10}P_{2}}}{_{20}P_{5}}$

This expression treats the cards within each hand as if their order matters. Reconsider whether the order you draw the cards changes the hand you end up holding.

3. $\dfrac{_{10}C_{3} \times {_{10}C_{2}}}{_{20}C_{5}}$

This expression multiplies the two favorable group combinations in the numerator and divides by the total combination for the full hand, which fits a situation where the order of the cards drawn does not matter.

4. $\dfrac{_{20}C_{5}}{_{10}C_{3} \times {_{10}C_{2}}}$

This expression places the total hand combinations in the numerator and the favorable group combinations in the denominator. Revisit which part of a probability fraction represents the entire sample space.

A debate coach is forming a 5-person panel with no distinct roles from 8 sophomores and 6 seniors. A student wants the probability of a panel with exactly 3 sophomores and 2 seniors and shows this work:

Total panels possible: ${}_{14}C_5 = 2002$

Ways to choose 3 sophomores from 8: ${}_{8}C_3 = 56$

Ways to choose 2 seniors from 6: ${}_{6}C_2 = 12$

Probability: $\dfrac{56 \times 12}{2002}$

One of these steps contains an error. Select the corrected probability, and select the step where the error occurred.

Select all that apply

1 $\dfrac{672}{2002}$

This value carries forward the same figure used for choosing 2 seniors from 6 in the work shown. Recompute that combination and trace how it changes the final fraction.

2 $\dfrac{840}{2002}$

This value comes from recomputing the combination for choosing 2 seniors from 6, then multiplying by the sophomore combination and dividing by the total panel combinations.

3 $\dfrac{840}{240240}$

This value changes the denominator to an arrangement of the 5 panel spots. Reconsider whether the panel assigns distinct roles to its members.

4 The error occurred while finding the total number of possible panels.

Recompute this step on its own and compare the figure shown in the work to what this step should produce.

5 The error occurred while finding the number of ways to choose 3 sophomores from 8.

Recompute this step on its own and compare the figure shown in the work to what this step should produce.

6 The error occurred while finding the number of ways to choose 2 seniors from 6.

Recomputing this step produces a different figure than the one shown in the work, which changes the final fraction once it is carried through.